USN

06CS42

Fourth Semester B.E. Degree Examination, Dec. 2013/Jan. 2014
Graph theory and Combinatorics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Let G = (V, E) be a connected undirected graph. What is the largest possible value for $|V|$ if $|E| = 19$ and $\deg(v) \geq 4$ for all $v \in V$? **(05 Marks)**

b. Define graph isomorphism. Show that the following two graphs are isomorphic. **(05 Marks)**

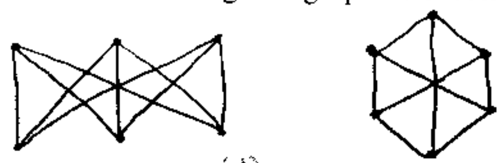

Fig. Q1(b)

c. Define : i) complete graph ii) Euler circuit iii) path. Give one example for each. **(05 Marks)**

d. Discuss Konigsberg bridge problem. **(05 Marks)**

2 a. Define Hamilton cycle. How many edge – disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles. **(06 Marks)**

b. If G is a connected simple planer graph with $n \geq 3$ vertices, $e > 2$ edges and r regions, then prove that i) $e \geq \frac{3}{2} r$ ii) $e \leq 3n - 6$ iii) $e \leq 2n - 4$ when G is triangle free. **(07 Marks)**

c. Define chromatic number. Find the chromatic polynomial for the cycle of length four. Hence find the chromatic number. **(07 Marks)**

3 a. Define a tree. In every tree T = (V, E), prove that, $|V| = |E| + 1$. **(06 Marks)**

b. Define : i) rooted tree ii) balanced tree. Draw all the spanning trees of the graph as shown in Fig. Q3(b). **(07 Marks)**

Fig. Q3(b)

c. Define prefix code. Obtain an optimal prefix code for message FALL OF THE WALL. Indicate the code. **(07 Marks)**

4 a. Define : i) cut –set ii) matching iii) complete matching. **(06 Marks)**

b. State Krushkal's algorithm. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph as shown in Fig. Q4(b). **(07 Marks)**

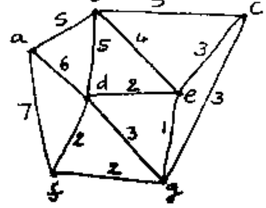

Fig. 4(b)

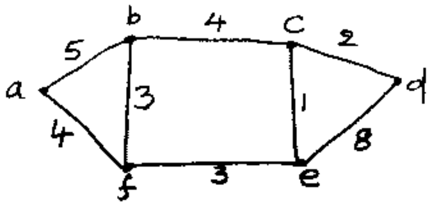

Fig. 4(c)

c. State max-flow and min-cut theorem. For the network shown below in Fig. Q4(c), find the capacities of all the cutsets between the vertices a and d, and hence determine the maximum flow between a and d. **(07 Marks)**

PART – B

5 a. Find the number of arrangements of the letters in TALLAHASSEE. How many of these arrangement have no adjacent A's. **(05 Marks)**

b. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with at least three spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? **(05 Marks)**

c. Determine the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$. **(05 Marks)**

d. Using the moves $R(x, y) \rightarrow (x + 1, y)$ and $U(x, y) \rightarrow (x, y + 1)$, find in how many ways can one go
   i) from (2, 1) to (7, 6) and not rise above the line $y = x - 1$
   ii) from (3, 8) to (10, 15) and not rise above the line $y = x + 5$. **(05 Marks)**

6 a. How many integers between 1 and 300 (inclusive) are
   i) Divisible by at least one of 5, 6. 8?
   ii) Divisible by none of 5, 6, 8? **(06 Marks)**

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. **(07 Marks)**

c. Find the rook polynomial for the 3 × 3 board by using the expansion formula. **(07 Marks)**

7 a. Using generating function, find the number of i) nonnegative ii) positive integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 28$. **(06 Marks)**

b. Use generating function to determine the number of four – element subsets of S = {1, 2, 3, ----- 15} that contain no consecutive integers. **(07 Marks)**

c. A company appoints 11 software engineers, each of whom is to be assigned to one of four offices of the company. Each office should get at least one of these engineers. In how many ways can these assignments be made? **(07 Marks)**

8 a. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(06 Marks)**

b. Solve the recurrence relation : $a_n + 4a_{n-1} + 4a_{n-2} = 8$ for $n \geq 2$ and $a_0 = 1$, $a_1 = 2$. **(07 Marks)**

c. Solve the recurrence relation $a_{n-2} - 2a_{n+1} + a_n = 2^n$, $n \geq 0$ and $a_0 = 1$, $a_1 = 2$ by the method of generating function. **(07 Marks)**

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